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Time : 65 minutes
Fall 2016-17.

MATHEMATICS 218
QUIZ I

NAME
ID#

Circle your section number :

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2 W	1 W	11 W	12 F	4 F	3 F	12 M	1 M	11M	4 M	3 M	11 Th	11 F	4 F	5 F

PROBLEM GRADE

PART I

1 12 / 14

2 14 / 14

3 16 / 16

4 8 / 8

PART II

5	6	7	8	9	10
a	a	a	a	a	a
b	b	b	b	b	b
c	c	c	c	c	c
d	d	d	d	d	d
e	e	e	e	e	e

5-10 30 / 30

PART III

11	12	13	14	15	16
T	T	T	T	T	T
F	F	F	F	F	F

11-16 18 / 18

TOTAL 98 / 100

PART I. Answer each of the following problems in the space provided for each problem (Problem 1 to Problem 4).

1. Use row echelon form of the following linear system to find the values of a and b for which the system:

$$x + 2y - z = 1$$
$$3x + 7y - az = 0$$
$$-2x - y + z = b$$

has

a. no solution
b. a unique solution
c. infinitely many solutions.

[14 points]

augmented matrix: $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 3 & 7 & -a & 0 \\ -2 & -1 & 1 & b \end{array}\right] \xrightarrow[-3R_1+R_2 \to R_2]{2R_1+R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 3-a & -3 \\ 0 & 3 & -1 & 2+b \end{array}\right]$

$\xrightarrow{-3R_2+R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & 3-a & -3 \\ 0 & 0 & -10+3a & 11+b \end{array}\right]$ ... Row echelon form

a) The system has no solution when its row echelon form is inconsistent, which means $-10+3a=0$ and $11+b \neq 0$

$a = \frac{10}{3}$ and $b \neq -11$

b) The system has unique solution when its row echelon form is consistent and ~~the~~ there is no zeros-row

$\Rightarrow$ for $-10+3a \neq 0$ and $11+b \neq 0$

$a \neq \frac{3}{10}$ and $b \neq -11$ (-2)

c) The system has infinitely many solutions when there is a redundant row of zeros and there is only 2 pivots for 2 rows $\Rightarrow$ for $-10+3a=0$ and $11+b=0$

$a = \frac{10}{3}$ and $b = -11$

2. Consider the matrix

$$A=\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & -1\\ 1 & 1 & 1\end{pmatrix},$$

(a) Find A^{-1}

[10 points]

$$\left[\begin{array}{ccc|ccc}1&0&1&1&0&0\\0&1&-1&0&1&0\\1&1&1&0&0&1\end{array}\right]\xrightarrow{-R_1+R_3\to R_3}\left[\begin{array}{ccc|ccc}1&0&1&1&0&0\\0&1&-1&0&1&0\\0&1&0&-1&0&1\end{array}\right]$$

$$\xrightarrow{R_2\leftrightarrow R_3}\left[\begin{array}{ccc|ccc}1&0&1&1&0&0\\0&1&0&-1&0&1\\0&1&-1&0&1&0\end{array}\right]\xrightarrow{R_3+R_1\to R_1}\left[\begin{array}{ccc|ccc}1&1&0&1&1&0\\0&1&0&-1&0&1\\0&1&-1&0&1&0\end{array}\right]$$

$$\xrightarrow[-R_2+R_1\to R_1]{-R_2+R_3\to R_3}\left[\begin{array}{ccc|ccc}1&0&0&2&1&-1\\0&1&0&-1&0&1\\0&0&-1&1&1&-1\end{array}\right]\xrightarrow{(-1)R_3}\left[\begin{array}{ccc|ccc}1&0&0&2&1&-1\\0&1&0&-1&0&1\\0&0&1&-1&-1&1\end{array}\right]$$

$$\underbrace{\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}}_{I}\qquad\underbrace{\begin{matrix}2&1&-1\\-1&0&1\\-1&-1&1\end{matrix}}_{A^{-1}}$$

So, $A^{-1}=\begin{pmatrix}2&1&-1\\-1&0&1\\-1&-1&1\end{pmatrix}$

(b) Use A^{-1} from part (a) above to find the solution $\mathbf{X}=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$ to the linear system $A\mathbf{X}=\mathbf{b}$,

where $\mathbf{b}=\begin{pmatrix}3\\5\\-2\end{pmatrix}$.

[4 points]

$AX=b$

$A^{-1}AX=A^{-1}b$

$IX=A^{-1}b$

$$\Rightarrow X=A^{-1}b=\begin{pmatrix}2&1&-1\\-1&0&1\\-1&-1&1\end{pmatrix}\begin{pmatrix}3\\5\\-2\end{pmatrix}=\begin{pmatrix}13\\-5\\-10\end{pmatrix}$$

3. Let A be 2×2 matrix such that $A^2+4A+4I=\mathbf{0}$

(a) Find A^{-1} in terms of A and I [8 points]

$$A^2+4A+4I=0$$

$$A^2+4A=-4I$$

$$A(A+4I)=-4I$$

$$A\left(\frac{A+4I}{-4}\right)=I$$

$$A\left(-\frac{1}{4}A-I\right)=I$$

So, $\boxed{A^{-1}=-\frac{1}{4}A-I}$ since when multiplied with A it gives the identity I.

(b) Suppose that $A=\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ such that $A^2+4A+4I=\mathbf{0}$. Find the values of a and b.

[8 points]

- $A^{-1}=-\frac{1}{4}A-I=-\frac{1}{4}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}-\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$=\begin{pmatrix} -\frac{a}{4}-1 & 0 \\ 0 & -\frac{b}{4}-1 \end{pmatrix}$$

- $AA^{-1}=I$

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}\begin{pmatrix} -\frac{a}{4}-1 & 0 \\ 0 & -\frac{b}{4}-1 \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -\frac{a^2}{4}-a & 0 \\ 0 & -\frac{b^2}{4}-b \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

So, $-\frac{a^2}{4}-a=1 \Rightarrow -\frac{a^2}{4}-a-1=0 \Rightarrow \boxed{a=-2}$

$-\frac{b^2}{4}-b=1 \Rightarrow -\frac{b^2}{4}-b-1=0 \Rightarrow \boxed{b=-2}$

4. If A and B are n×n matrices such that $A^2 = I, B^2 = I,$ *and* $(AB)^2 = I$. Prove that AB=BA

[8 points]

$(AB)^2 = I$

$ABAB = I$

$ABABB = IB$

$ABAB^2 = B$

$ABAI = B$

$ABA = B$

$AABA = AB$

$A^2BA = AB$

$IBA = AB$

So, $BA = AB$ ... proved!

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PART II. Circle the correct answer for each of the following multiple choice problems (Problem 5 to Problem 10) IN THE TABLE IN THE FRONT PAGE. [5 points for each correct answer].

5. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$. Then

(a) $A^{10} = I$
(b) $A^3 = I$ (identity matrix)
(c) $A^2 = \mathbf{O}$ (zero matrix)
(d) A is not invertible
(e) none of the above

[5 points]

6. Let A be an invertible n×n matrix. Which **one** of the following statements is **FALSE**:

(a) AB is invertible for any n×n matrix B.
(b) The number of nonzero rows in a row echelon form of A is n.
(c) A^t is invertible.
(d) det (A) ≠0.
(e) The reduced row echelon form of A is I.

[5 points]

7. If $A=\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ such that $|A|=5$, then $\begin{vmatrix} g & h & i \\ 6a+d & 6b+e & 6c+f \\ 2a & 2b & 2c \end{vmatrix}$ is equal to:

(a) -60
(b) -30
(c) 60
(d) -10
(e) none of the above

[5 points]

8. Let A be an n×n matrix. Which **one** of the following statements is **FALSE:**

?

(a) If the reduced row echelon form of A is I, then A is invertible
(b) If the reduced row echelon form of A is **not** I, then det(A)=0
(c) If the homogeneous matrix equation AX=**0** has only the trivial solution, then A is invertible
(d) If A is not invertible, then the matrix equation AX=**b** has infinitely many solutions for all **b**.
(e) If A is invertible, then the homogeneous matrix equation AX=**0** has only the trivial solution

[5 points]

9. The value of the number k for which the matrix

$$A=\begin{pmatrix} 1 & 6 & 3 \\ 1 & k & 1 \\ 0 & k & 2 \end{pmatrix},$$

is **<u>not</u>** invertible is:

(a) k= 1
(b) k= 3
(c) k= 2
(d) k= 0
(e) none of the above

[5 points]

10. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$, and $\mathbf{v}_5$ be vectors in R^n such that $\mathbf{v}_3 = \mathbf{v}_1 + \mathbf{v}_2$, and $\mathbf{v}_5 = \mathbf{v}_3 + \mathbf{v}_4$. Which **one** of the following statements is **<u>FALSE</u>:**

(a) $\mathbf{v}_1$ is a linear combination of $\mathbf{v}_4$ and $\mathbf{v}_5$
(b) $\mathbf{v}_2$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_3$
(c) $\mathbf{v}_5$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_4$
(d) $\mathbf{v}_3$ is a linear combination of $\mathbf{v}_4$ and $\mathbf{v}_5$
(e) $\mathbf{v}_3$ is a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$

[5 points]

PART III. Answer **TRUE** or **FALSE** only, **IN THE TABLE IN THE FRONT PAGE (3 points for each correct answer)**

11. Every diagonal matrix is invertible.

12. If A is any square $n \times n$ matrix, then $A^t A$ is symmetric

13. If A is a 3×3 matrix such that $\det(2A^{-1}) = 4$, then $\det(A^t) = 2$

14. If A and B are $n \times n$ matrices such that A is invertible and $(A^{-1}B)^t$ is invertible, then B is invertible.

15. The vector $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ is a linear combination of the vectors $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

16. If A and B are $n \times n$ matrices such that $AB=0$ and $A \neq 0$, then $B=0$.

[18 points]